Techniques of Integration

Note: $f(x)$ and $g(x)$ are functions of x, $\int f'(x)dx = f(x) + c$ and $\int g'(x)dx = g(x) + c$ and c represents an arbitrary constant

Basic Integration

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Remember: Power plus one, divided by final power

$$\int [f(x)]^n \, dx = \frac{[f(x)]^{n+1}}{(n+1)f'(x)} + c \ , n \neq 1$$

Exmaple:

$$\int (2x+3)^4 \, dx = \frac{(2x+3)^5}{(5)\frac{d}{dx}(2x+3)} + c = \frac{(2x+3)^5}{10} + c$$

Logarithmic Functions

$$\int \frac{f'(x)}{f(x)} \, dx = \ln f(x) + c$$

Example:

$$\int \frac{4x+2}{2x^2+2x-1} \, dx = \ln(2x^2+2x-1) + c$$

$$\int \frac{x+1}{2x^2+2x+1} \, dx = \frac{1}{2}\int \frac{2x+2}{2x^2+2x+1} \, dx = \frac{1}{2}\ln(2x^2+2x+1) + c$$

Exponential Functions

$$\int f'(x)e^{f(x)} \, dx = e^{f(x)} + c$$

Example:

$$\int (2x+1)e^{(x^2+x)} \, dx = e^{(x^2+x)} + c$$

$$\int e^{2x+1} \, dx = \frac{1}{2}\int 2e^{2x+1} \, dx = \frac{1}{2}e^{2x+1} + c$$

Laws of Integration

$$\int f(x) \pm g(x) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int k \, f(x) \, dx = k \int f(x) \, dx$$

Definite Integrals

$$\int_b^a f(x) \, dx = -\int_a^b f(x) \, dx$$

Exmaple:

$$\int_1^3 (2x+1) \, dx = -\int_3^1 (2x+1) \, dx$$

$$\int_b^a f(x) \, dx = \int_c^a f(x) \, dx + \int_b^c f(x) \, dx$$

Example:

$$\int_1^3 (2x+1) \, dx = \int_2^3 (2x+1) \, dx + \int_1^2 (2x+1) \, dx$$

Trigonometric Functions

$$\int \sin f(x) \, dx = \frac{-\cos f(x)}{f'(x)} + c$$

$$\int \cos f(x) \, dx = \frac{\sin f(x)}{f'(x)} + c$$

$$\int \sec^2 f(x) \, dx = \frac{\tan f(x)}{f'(x)} + c$$

Note: $f(x)$ is a linear function

Example:

$$\int \sin(2x+1) \, dx = \frac{-\cos(2x+1)}{\frac{d}{dx}(2x+1)} + c = \frac{-\cos(2x+1)}{2} + c$$

$$\int \cos(1-4x) \, dx = \frac{\sin(1-4x)}{\frac{d}{dx}(1-4x)} + c = \frac{\sin(1-4x)}{-4} + c$$